

cut out of the rolling plane. A more quantitative check on the theory is difficult because not enough is known about the coefficients  $K_{LF}$  and  $K_{SC}$ . As discussed by Chikazumi *et al.*, parameters such as  $p_0$  and  $p'$  are not very accessible to experimentation as they depend sensitively on the details of dislocation motion. In addition, the order parameters  $s$  and  $\sigma$  are expected to decrease with increasing deformation,<sup>24,25</sup> so that the integrated forms of Eqs. (2) and (3) may be required. Attempt at a quantitative study was made by CSI, but the results were inconclusive for the above reasons.

Regarding the effect of long- and short-range-order on the slip-induced anisotropy, cases (1), (4), (5), and (7) for rolling in Table II are of interest, since different directions of the easy axis are predicted depending on the type of order. At compositions close to 75% Ni, appropriate heat treatments may be used to bring about mainly one type of ordering. Or, short-range order may be achieved at the expense of long range by choosing the composition well away from the 75% Ni region. While the kinetics of long-range ordering is sluggish, short-range order appears unavoidable over a range of the Fe-Ni composition.<sup>26</sup> Although the above analyses were made for 75% Ni-25% Fe only, a change in composition would affect the number of induced atom pairs (hence  $K_{LF}$  and  $K_{SC}$ ) but not their directions. Consequently the predicted positions of the easy directions outlined in Table II would still be applicable.

It is assumed in the present analysis that the choice of the operating slip systems is made on the basis of satisfying the macroscopic stress and strain conditions. Wire drawing has been considered as tension along the wire axis and rolling as a triaxial stress system with the symmetry directions as principal stress axes. In the two cases ((110)  $[\bar{1}\bar{1}2]$  and (112)  $[\bar{1}\bar{1}0]$ ) where the operating slip systems based on stress consideration do not satisfy strain compatibility, alternative analyses were made to provide for additional slip systems.

It is further assumed in the present analysis that all operating slip systems operate homogeneously throughout the sample. If certain systems exclude one another, for reasons such as unequal hardening and change in stress due to lattice reorientation during deformation, the resulting anisotropies may be different from those of Table II. In the CSI study, for example, systems (1), (2), (4), and (5) are predicted in (001)  $[\bar{1}10]$  rolling. Slip-line observations, however, show that systems (1) and (2) operate on the top side of the crystal and (4) and (5) on the bottom. Hence inhomogeneity of deformation may have to be considered in individual cases.

Aside from the above considerations, Table II presents several interesting conclusions. During wire

drawing, for example, the wire axis becomes magnetically hard or easy depending on whether the orientation is  $\langle 001 \rangle$  or  $\langle 111 \rangle$ . As for plane strain deformation, usually realized in rolling, most of the orientations predict the rolling direction as the induced easy axis, notable exceptions being (001)  $[\bar{1}00]$  and (110)  $[\bar{0}01]$ . The case of (110)  $[\bar{1}\bar{1}2]$  rolling is also interesting since it predicts an easy axis other than the rolling or transverse direction, hence the theory can be put to a severe test here.

Table II also has implications for rolling polycrystalline materials. Since all the orientations described are symmetrical orientations (with respect to rolling), an initially randomly oriented grain probably rotates to one of these orientations after a small deformation. A majority of these orientations place the easy axis along the rolling direction; which may explain the observation that the rolling direction is the easy axis for an initially random polycrystalline aggregate of Permalloy.<sup>2</sup> In this connection, it is interesting to note that the rolling direction continues to be the easy axis at 90% thickness reduction or more where the texture is no longer random. The rolled texture in face-centered cubic Fe-Ni alloys can be described as (112)  $[\bar{1}\bar{1}\bar{1}]$  plus (110)  $[\bar{1}\bar{1}2]$ , the former component being the larger.<sup>2</sup> Table II reveals that rolling of a (112)  $[\bar{1}\bar{1}\bar{1}]$  crystal results in an easy axis along the rolling direction. As for the (110)  $[\bar{1}\bar{1}2]$  texture, it actually consists of two symmetrical components, which may be written as (110)  $[\bar{1}\bar{1}2] + (110) [\bar{1}\bar{1}\bar{2}]$ . Hence the anisotropy listed in Table II for (110)  $[\bar{1}\bar{1}2]$  should be superposed onto that for (110)  $[\bar{1}\bar{1}\bar{2}]$ . When this is done, the effective easy axis is again most likely along the rolling direction.

Finally it should be noted that plane strain conditions can be approximated by processes other than rolling. Two cases of technological importance are wire flattening by rolling and flat-drawing. These two methods are employed in the fabrication of thin magnetic tapes of narrow width for memory applications. During roll-flattening, the material flows laterally without much elongation. The flat-drawing process, on the other hand, results in axial elongation without much change in the lateral direction. Such processing techniques, as preliminary studies have shown,<sup>27</sup> are expected to produce unusual textures and magnetic anisotropies. The results of the experimental studies involving rolling and wire drawing of single crystals to test the theoretical analyses presented here as well as the results of flattening polycrystalline wire materials will be presented in a subsequent paper.

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<sup>24</sup> P. S. Rudman and B. L. Averbach, *Acta Met.* 4, 382 (1956).  
<sup>25</sup> J. B. Cohen and M. B. Bever, *Trans. Met. Soc. AIME* 218, 155 (1960).

<sup>26</sup> M. F. Collins, R. V. Jones, and R. D. Lowde, *J. Phys. Soc. Japan* 17, Suppl. B-III, 19 (1962).

<sup>27</sup> G. Y. Chin, L. L. Vanskike, and H. L. Andrews, *J. Appl. Phys. Suppl.* 35, 867 (1964).



## APPENDIX

## Relation between Glide-Shear, Slip Density, and Macrostrain

The "slip density"  $S$ , as defined in the CSI analysis, is the average or effective number of dislocations passed per atomic (slip) plane. The glide-shear due to slip can thus be expressed as  $\gamma = Sb/d$ , where  $b$  is the strength of the Burger's vector and  $d$  the slip plane spacing. For  $\{111\}$   $\langle 110 \rangle$  slip in face-centered cubic materials,  $b = a/\sqrt{2}$  and  $d = a/\sqrt{3}$  ( $a$  = lattice constant). Hence

$$\gamma = S\left(\frac{3}{2}\right)^{\frac{1}{2}}. \quad (\text{A1})$$

Now the glide shear is related to the macroscopic strain tensor components by the equations<sup>13</sup>

$$\begin{aligned} \epsilon_{xx} &= \gamma n_x d_x, \\ \epsilon_{yy} &= \gamma n_y d_y, \\ \epsilon_{zz} &= \gamma n_z d_z, \\ \epsilon_{yz} &= (\gamma/2)(n_y d_z + n_z d_y), \\ \epsilon_{zx} &= (\gamma/2)(n_z d_x + n_x d_z), \\ \epsilon_{xy} &= (\gamma/2)(n_x d_y + n_y d_x), \end{aligned} \quad (\text{A2})$$

where  $n_x, n_y, n_z$  are components along the cubic coordinates of a unit vector normal to the slip plane and  $d_x, d_y, d_z$  are the components of a unit vector along the slip direction. As an example, take slip on the (111)  $[0\bar{1}1]$  system [No. (1) in Table I],  $n_x = n_y = n_z = 1/\sqrt{3}$ ,  $d_x = 0$ ,  $d_y = -1/\sqrt{2}$ ,  $d_z = 1/\sqrt{2}$ . From Eqs. (A2), we get

$$\begin{aligned} \epsilon_{xx} &= 0, \\ \epsilon_{yy} &= -(\gamma/\sqrt{6}), \\ \epsilon_{zz} &= \gamma/\sqrt{6}, \\ \epsilon_{yz} &= 0, \\ \epsilon_{zx} &= (\gamma/2) \cdot (1/\sqrt{6}), \\ \epsilon_{xy} &= -(\gamma/2)(1/\sqrt{6}). \end{aligned} \quad (\text{A3})$$

Finally, by converting  $\gamma$  to  $S$  via Eq. (A1),

$$\begin{aligned} \epsilon_{xx} &= 0, \\ \epsilon_{yy} &= -(S_1/2), \\ \epsilon_{zz} &= (S_1/2), \\ \epsilon_{yz} &= 0, \\ \epsilon_{zx} &= S_1/4, \\ \epsilon_{xy} &= -(S_1/4), \end{aligned} \quad (\text{A4})$$

which are the values entered in Table I.